Name: ________________

Ms. Torres **Per: ___**

A.P. CALCULUS BC

Summer Calculus - Readiness Packet

Instructions:

1. This summer review packet contains worksheets reviewing topics necessary for Calculus. Topics are from
   - Arithmetic
   - Algebra 1
   - Geometry
   - Algebra II
   - Trigonometry
   - Calculus
2. This summer review packet is **due on the first day of class** of the fall semester, counts as homework, and **is required to be admitted into the A.P. Calculus class**.
3. You may NOT use a calculator before the "Quadratic Formula" worksheet. (**Show all work whenever there is space; otherwise, no credit.)**
4. Estimated time to do all of this work is 10 hours.
5. We will start with Calculus on the first day of school.

Materials required for fall semester

On the first day of school you will need the following materials:

1. Textbook: Rogawski's *Calculus for AP* - Early Transcendentals,* 2nd Edition (2012) ISBN -13: 978-1-4292-5074-0 (has a red strip on the right side of the cover)
2. Calculator: A non-intercommunicating scientific calculator (graphing capability required) (such as a TI-83/84)
3. Binder(s) for notes and homework assignments (organized and with extra paper)
4. 2 pencils & an eraser (no pen)

Decimal Practice

dd/Suma.

) 1.39	(2) 11.94	(3) 7.91	(4) 4.9	(5) 6.92
+ 2.4	+ 2.71	+ 3.05	+ 3.71	+ 3.1

btract/Resta.

) 2.4	(7) 11.94	(8) 7.91	(9) 4.9	(10) 6.92
− 1.39	− 2.71	− 3.05	− 3.71	− 3.1

ultiply/Multiplica.

) 2.4	(12) 5.61	(13) 11.9	(14) 5.4	(15) 3.91
x 8	x 9	x 6	x 3.2	x .32

ivide/Divide.

) $8\overline{)3}$	(17) $7\overline{)2}$	(18) $.3\overline{)1}$	(19) $11\overline{)155}$	(20) $5\overline{)31.2}$

Adapted from Mark Burniston

Fraction Basics I

Name: ________
Per: __

Make the indicated equivalent fraction.

(1) $\frac{1 \times 2}{2 \times 2} = \boxed{\frac{2}{4}}$ (2) $\frac{1}{2} = \frac{}{8}$ (3) $\frac{1}{4} = \frac{}{8}$ (4) $\frac{2}{3} = \frac{}{12}$

Reduce each fraction completely.

(5) $\frac{2 \div 2}{4 \div 2} = \boxed{\frac{1}{2}}$ (6) $\frac{4}{8} = $ ___ (7) $\frac{2}{8} = $ ___ (8) $\frac{8}{12} = $ ___

Convert to an improper fraction.

(9) $1\frac{+1}{\times 2} = \frac{}{2}$ (10) $2\frac{1}{4} = \frac{}{4}$ (11) $3\frac{1}{3} = \frac{}{3}$ (12) $4\frac{5}{8} = \frac{}{8}$

Convert to a mixed number. (÷)

(13) $\frac{3}{2}$ (14) $\frac{9}{4}$ (15) $\frac{16}{3}$ (16) $\frac{31}{8}$

Add.

(17) $\frac{1}{3} + \frac{1}{3}$ (18) $\frac{1}{4} + \frac{1}{4}$ (19) $\frac{1}{5} + \frac{2}{5}$ (20) $\frac{3}{8} + \frac{4}{8}$

Subtract.

(21) $\frac{1}{3} - \frac{1}{3}$ (22) $\frac{2}{4} - \frac{1}{4}$ (23) $\frac{3}{5} - \frac{1}{5}$ (24) $\frac{7}{8} - \frac{4}{8}$

Adapted from Mark Burniston

fractions Practice I

Add/Suma

① $\frac{5}{7}+\frac{1}{7}=\boxed{\frac{6}{7}}$ ② $\frac{8}{15}+\frac{5}{15}=$ ③ $\frac{17}{25}+\frac{4}{25}=$

④ $\frac{7}{10}+\frac{2}{10}=$ ⑤ $\frac{4}{11}+\frac{3}{11}=$ ⑥ $\frac{5}{8}+\frac{2}{8}=$

Subtract/Resta

⑦ $\frac{2}{4}-\frac{1}{4}=\boxed{\frac{1}{4}}$ ⑧ $\frac{2}{5}-\frac{1}{5}=$ ⑨ $\frac{4}{11}-\frac{3}{11}=$

⑩ $\frac{5}{12}-\frac{3}{12}=$ ⑪ $\frac{7}{18}-\frac{4}{18}=$ ⑫ $\frac{17}{25}-\frac{4}{25}=$

fractions Practice II

Add: utilize equivalent fractions to first get same denominator.

① $\frac{1^{\times 2}}{2_{\times 2}}+\frac{1}{4}$ ② $\frac{1}{2}+\frac{1}{8}=$ ③ $\frac{7}{10}+\frac{1}{5}=$

$=\frac{2}{4}+\frac{1}{4}=\boxed{\frac{3}{4}}$

Subtract: Utilize equivalent fractions to first get same denominators.

④ $\frac{1^{\times 2}}{2_{\times 2}}-\frac{1}{4}=$ ⑤ $2\frac{1}{2}-1\frac{5}{8}=$ ⑥ $\frac{7}{10}-\frac{2}{5}=$

$\frac{2}{4}-\frac{1}{4}=\boxed{\frac{1}{4}}$

Adapted from Mark Burniston

$\times$, $\div$ Fractions

Name: ____________
Per: ___

Multiply.

(1) $\frac{3}{4} \times \frac{1}{2} = \boxed{\frac{3}{8}}$ (2) $\frac{1}{3} \times \frac{1}{5}$ (3) $\frac{1}{8} \times \frac{1}{2}$ (4) $\frac{4}{5} \times \frac{1}{2}$

(5) $\frac{3}{4} \times \frac{1}{3}$ (6) $\frac{5}{8} \times \frac{1}{4}$ (7) $\frac{8}{9} \times \frac{1}{3}$ (8) $\frac{7}{10} \times \frac{1}{8}$

(9) $\frac{7}{10} \times \frac{2}{5}$ (10) $\frac{3}{5} \times \frac{7}{8}$ (11) $\frac{1}{2} \times \frac{1}{4}$ (12) $\frac{1}{4} \times \frac{1}{4}$

(13) $\frac{5}{6} \times 1\frac{1}{2}$ (14) $1\frac{2}{3} \times \frac{4}{5}$ (15) $1\frac{2}{5} \times 2\frac{1}{3}$ (16) $1\frac{3}{4} \times 3\frac{1}{2}$

$= \frac{5}{6} \times \frac{3}{2} = \boxed{\frac{15}{12}}$.

Divide.

(1) $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \boxed{\frac{9}{8}}$. (2) $\frac{1}{3} \div \frac{1}{5}$ (3) $\frac{1}{8} \div \frac{1}{2}$ (4) $\frac{4}{5} \div \frac{1}{2}$

(5) $\frac{3}{4} \div \frac{1}{3}$ (6) $\frac{5}{8} \div \frac{1}{4}$ (7) $\frac{8}{9} \div \frac{1}{3}$ (8) $\frac{7}{10} \div \frac{1}{8}$

(9) $\frac{7}{10} \div \frac{2}{5}$ (10) $\frac{3}{5} \div \frac{7}{8}$ (11) $\frac{1}{2} \div \frac{1}{4}$ (12) $\frac{1}{4} \div \frac{1}{4}$

(13) $\frac{5}{6} \div 1\frac{1}{2}$ (14) $1\frac{2}{3} \div \frac{4}{5}$ (15) $1\frac{2}{5} \div 2\frac{1}{3}$ (16) $1\frac{3}{4} \div 3\frac{1}{2}$

Adapted from Mark Burniston

Integer Review

Name: ____________
Per: ___

Hint: (+) = ↑, (−) = ↓.

Hint: Add Opposite.

1. $(+8) + (+6) =$
2. $(-8) + (-6) =$
3. $(+8) + (-6) =$
4. $(-8) + (+6) =$
5. $(-37) + (+24) =$
6. $(+7) - (+5) =$
7. $(-7) - (-5) =$
8. $(+7) - (-5) =$
9. $(-7) - (+5) =$
10. $(-37) - (-24) =$

---

11. $(+12) \times (+3) =$
12. $(-12) \times (-3) =$
13. $(+12) \times (-3) =$
14. $(-12) \times (+3) =$
15. $(+24) \times (-3) =$
16. $(+12) \div (+3) =$
17. $(-12) \div (-3) =$
18. $(+12) \div (-3) =$
19. $(-12) \div (-3) =$
20. $(-24) \div (+3) =$

Adapted from Mark Burniston

Name: ____________

Per: ____

Order of Operations VII

+, −, x, ÷ must be done in the following order:

(1.)	**P**arentheses	(innermost first)
(2.)	**E**xponents	(L→R)
(3.)	**M**ultiplication & **D**ivision	(L→R)
(4)	**A**ddition & **S**ubtraction	(L→R).

(If not, you will get the wrong answer)!

(1) $2 + 3 \times 9 - 4$

$= \boxed{}$

(2) $3 \times 5 + 4 \div 2$

$= \boxed{}$

(3) $7 + 9 \times 3 \div 3 - 1$

$= \boxed{}$

(4.) $2 + 3^2 - 1 \times 5$

$= \boxed{}$

(5) $(5+3) \times 2 + 3^2$

$= \boxed{25}$

(6) $(6+2^2) - 4 \times 2$

$= \boxed{2}$

(7.) $5^2 + 9 \times (3) - 1$

$= \boxed{51}$

(8.) $(3^2+6) \div 3 \times 5 - 1$

$= \boxed{24}$

Order of Operations IV

Make sure you get the following answers!

1) $42 - 9 \times 3 + 16$ $\boxed{31}$ 2 $42 - 9 \times 3 + 16 \div 4$ $\boxed{19}$

) $42 - (2 \times 3)^2 + 16 \div 4$ $\boxed{10}$ 4 $42 - (2+3)^2 + 16 \div 4$ $\boxed{21}$

) $42 - 2 \times 3^2 + 16 \div 4$ $\boxed{28}$ 6 $42 - 2 \times (3^2 + 16) \div 5$ $\boxed{32}$

) $42 - 2 \times 3^2 + (15 \div 5)$ $\boxed{27}$ 8 $(42-2) \times 3 + 15 \div 5$ $\boxed{123}$

) $[(42-2) \times 3 + 15] \div 5$ $\boxed{27}$ 10 $(42-9) \times [(16-4) \div 3]$ $\boxed{132}$

) $9^2 + 4 \div 2 - 7$ $\boxed{76}$ 12 $(9^2+4) \div 5 - 7$ $\boxed{10}$

) $(9^2+4) \div 5 + 3$ $\boxed{20}$ 14 $[(9^2+4) \div 5 + 3] \div 5$ $\boxed{4}$

Summary of Arithmetic

Whole Numbers:

Line up numbers by their Right Sides.

+ : Greater than or equal to 10 => Carry over to next column (to the left).

- : Top number Smaller than Bottom number => Borrow (from column to the left: "1 less, 10 more").

x : Multiply One's digit in bottom # by Every digit in the top # (= 1st Row);
Repeat, using Ten's digit in bottom # (= 2nd Row & moved over 1 column)
Repeat for each digit in bottom #; then Add up all Rows.

/ : Try dividing divisor into 1st digit of dividend; if too big try 1st 2 digits; if still too big, try 1st 3 digits, etc.
Write Quotient and Multiply it with Divisor & Subtract it from LEFT side of dividend.
Bring down Next Digit of Dividend & Repeat process.

Decimals:

+ , - : Decimal Points need to be in-line.

x : The # of Dec.Places in the problem = The # of Dec.Places in the Answer. (Count them.)

/ : Move the Decimal Point in the Divisor all the way to the Right, &
Move the Decimal Point in the Dividend to the Right the Same # of places.

Fractions:

+ , - : Be certain Denominators are the Same first!

x : Multiply Numerators, Multiply Denominators. (Regardless of Denominators).

/ : Multiply by the Reciprocal (upside-down) of the 2nd Fraction.

Integers:

Integers are Positive (+) or Negative (-) Whole numbers.

+ : (+) indicates a direction of "Higher", & (-) indicates a direction of "Lower".

- : To Subtract (-), Add (+) the OPPOSITE!

x & / : (+) x (+) = (+). (-) x (-) = (+). But (+) x (-) = (-).

Order of Operations:

Operations must be done in "Please Excuse My Dear Aunt Sally" order:

( ) : Parentheses (operations inside them)

exp : Exponents (= repeated multiplication)

x , / : Multiply & Divide (Left --> Right)

+ , - : Add & Subtract (Left --> Right)

Adapted from Mark Burniston

Name: ______________

Per: ___

BASIC ALGEBRA

Def^n ARITHMETIC IS MATH WITH KNOWN #S.

Def^n ALGEBRA IS MATH WITH UNKNOWN #S.
(HOPEFULLY, THEIR VALUES WILL BE KNOWN LATER.)
(GIVEN AN EQUATION, WE CAN FREQUENTLY FIND X.)

Def^n CONSTANT: ANYTHING (SUCH AS #S) THAT DOESN'T CHANGE.

a) KNOWN #S such as $1, 2, 3, -5, \frac{-1}{7}, 3.14$, etc.

b) UNKNOWN #S (& constant) indicated by "a", "b", "c", etc.

Def^n VARIABLE: ANYTHING (SUCH AS #S) THAT DOES CHANGE.

a) INDEPENDENT: The variable quantity that you're starting with.

ex1 x, t, u, etc.

b) DEPENDENT: A variable quantity that is related to and dependent on the independent variable's value.
(Also called a variable expression, which is a list of operations to be performed on the initial quantity (i.e. a related #).)

ex1 y, T, z, $f(x)$, $g(x)$, $h(x) = x^2 + 1$.

(i) FUNCTION: A special (and the most common) type of dependent variable/expression that generates exactly 1 output value for each value of the independent variable put into it['s formula].

Evaluate:
- ex1 If $x = 5$, $y = 2x + 1 = \boxed{}$
- ex2 If $x = 10$, $f(x) = x^2 - 1 = \boxed{}$

Like Terms Name: ________

Per: ____

Circle the two similar monomials in each set. Then write their sum on the right. (by adding coefficients)

					SUM
1	$6x^3$	$5x^2$	$2x$	$3x^2$	
2	$3x^2y$	$4x$	$7xy$	$5x^2y$	
3	$2a^2b^3$	$8b$	$12a^2b^3$	$9a^3$	
4	$13n$	$6mn$	$8mn$	$4m^2$	
5	$7pq$	5	$12pq^2$	8	
6	$9s^3t$	$4st^2$	$3s^2t$	$2s^3t$	
7	$5u^4v^2w$	$2u^3vw^2$	$8u^2v$	$4u^3vw^2$	
8	$7h^2k^3$	$4hk^3$	$15h^2k$	$9h^2k$	
9	$3rs^2$	$8r^2s$	rs^2	$2r^3$	
10	$2abc$	12	$5xy^2$	9	

Simplifying Expressions

Simplify by combining the similar monomials ("like terms")

1. $2x - 5 + x + 3$ $\boxed{}$
2. $4m - 3 + 3m + 1$ $\boxed{}$
3. $3x^2 - 2x + 1 + x^2 - 2x + 3$ $\boxed{}$
4. $3y^2 - 5 + 2y^2 - 3y + 4$ $\boxed{}$
5. $a^2 + 3ab - 4ab + 3a^2$ $\boxed{}$
6. $8p^2 - 5pq + 6q^2 - 2p^2 + 3pq - 4q^2$ $\boxed{}$
7. $4 - 3n - 5n^2 - 2 + n - 3n^2$ $\boxed{}$
8. $a^2 + ab - ab - b^2$ $\boxed{}$
9. $5x - 3t - 7 - x + 2t + 3$ $\boxed{}$
10. $u^3 - 3u^2v + 2uv^2 + 3u^2v - 2uv^2 - v^3$ $\boxed{}$

Adapted from Mark Burniston

Solving Equations I (1-step)

Name: ________ Per: ___

Addition Equations.

① $x + \cancel{2} = 5$
$\quad -\cancel{2} \quad -2$
$\boxed{x = 3}$

② $x + 3 = 9$ ③ $x + 5 = 47$

④ $x + 14 = 23$ ⑤ $x + 57 = 105$ ⑥ $x + 215 = 891$

Subtraction Equations.

① $x - \cancel{2} = 5$
$\quad +\cancel{2} \quad +2$
$\boxed{x = 7}$

② $x - 3 = 9$ ③ $x - 5 = 47$

④ $x - 14 = 23$ ⑤ $x - 57 = 105$ ⑥ $x - 215 = 891$

Multiplication Equations.

① $\frac{\cancel{2}x}{\cancel{2}} = \frac{10}{2}$
$\boxed{x = 5}$

② $5x = 15$ ③ $3x = 30$

④ $6x = 48$ ⑤ $12x = 96$ ⑥ $25x = 875$

Division Equations.

① $(\cancel{2}) \cdot \frac{x}{\cancel{2}} = 10 \cdot (2)$
$\boxed{x = 20}$

② $\frac{x}{5} = 15$ ③ $\frac{x}{3} = 30$

④ $\frac{x}{6} = 48$ ⑤ $\frac{x}{12} = 96$ ⑥ $\frac{x}{25} = 875$

Slope II

Name: ______

Per: ___

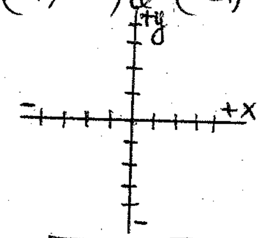
*Recall: Slope is a ratio (fraction) between 2 quantities (numbers):

$$\text{Slope} = \frac{\text{rise}(\uparrow)}{\text{run}(\leftrightarrow)} = \frac{\text{up}}{\text{over}} = \text{"}m\text{"}.$$

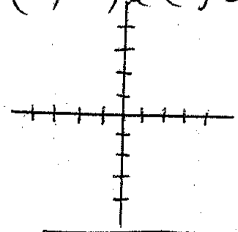
A. Graphing Slope through 2 Points

Plot the 2 points given & determine the slope between them.

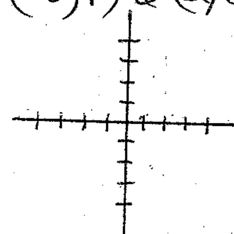
1. $(1,2)$ & $(-2,-1)$ 2. $(1,2)$ & $(-1,-2)$ 3. $(-2,1)$ & $(2,3)$



$m =$



$m =$



$m =$

B. Calculating Slope

Find the slope between points $P_1(x_1,y_1)$ & $P_2(x_2,y_2)$ using $\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$.

4. $(4,2)$ & $(2,4)$ 5. $(-4,2)$ & $(2,4)$ 6. $(-6,0)$ & $(0,6)$

$m =$ $m =$ $m =$

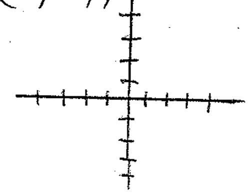
7. $(4,5)$ & $(7,3)$ 8. $(1,-1)$ & $(1,5)$ 9. $(-1,2)$ & $(-1,4)$

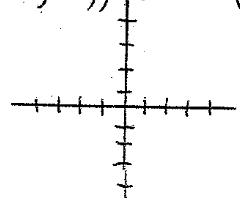
$m =$ $m =$ $m =$

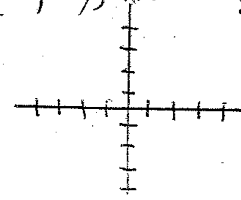
C. Graphing Slope through 1 Point

Draw the line through the given point with the given slope.

10. $(1,2)$, $m = \frac{1}{2}$ 11. $(-2,3)$, $m = -2$ 12. $(-1,2)$, $m = -\frac{2}{3}$







Adapted from Mark Burniston

GRAPHING III (Point-Slope form)

Name: ________

Per: ___

To find the equation of a line when given a point (x_1, y_1) and a slope (m), use "Point-Slope" form.

Point-Slope form is: $\boxed{y - y_1 = m(x - x_1)}$.

ex/ Find the equation of the line through (3, 4) with slope = 5

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 5(x - 3)$$
$$y - \cancel{4} = 5x - 15$$
$$\cancel{+4} \qquad + 4$$
$$EQ_{line} = \boxed{y = 5x - 11}$$

A. EQUATION OF LINE GIVEN POINT & SLOPE

Find the $y = mx + b$ equation of each line.

1. (2, 3) & m = 4
2. (5, 2) & m = ⁻3
3. (1, 6) & m = 7

y = ______ y = ______ y = ______

4. (⁻5, 8) & m = 6
5. (⁻4, ⁻1) & m = $\frac{1}{2}$
6. (⁻7, 5) & m = 0

y = ______ y = ______ y = ______

B. EQUATION OF LINE GIVEN 2 POINTS

Find the $y = mx + b$ equation of each line. First use $\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$.

7. (2, ⁻1) & (⁻3, 14)
8. (2, 3) & (⁻1, 1)

Graphing Review

A. Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Find the slope of the line between the given points.

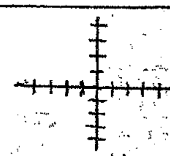
1. $(1, 4)$ & $(-2, -2)$ $m =$
2. $(-2, 7)$ & $(1, -2)$ $m =$
3. $(-5, -4)$ & $(1, -1)$ $m =$
4. $(3, -2)$ & $(-1, -2)$ $m =$

B. Slope-Intercept Form: $y = mx + b$

5. $y = 2x - 3$

$m =$

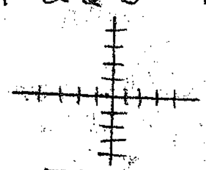
$b =$



6. $m = 4,\ b = -7$

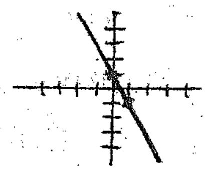
$EQ_{line} =$ $y =$

7. Graph the line with $m = 2$ & $b = -1$.



$EQ_{line} =$ $y =$

8.



$m =$

$b =$

$EQ_{line} =$ $y =$

C. Point-Slope Form: $y - y_1 = m(x - x_1)$

Find the equation of the line from the given information.

9. $(6, 3)$ & $m = -\frac{1}{3}$

$y =$

10. $(5, -7)$ & $(3, -1)$

$y =$

D. Standard Form: $Ax + By = C$

11. Write in Standard form: $5 + 2y = 7x$
12. Write in Slope-Intercept form $3x + 2y = 5$

EXPONENT RULES

Name:
Per: ___

① $\boxed{X^m \cdot X^n = X^{m+n}}$

② $\boxed{(X^m)^n = X^{mn}}$

③ $\boxed{\dfrac{X^m}{X^n} = X^{m-n}}$

④ $\boxed{X^0 = 1}$

⑤ $\boxed{X^{-m} = \dfrac{1}{X^{+m}}}$

① $8^0 =$ $\square$

② $7^2 \cdot 7^3 =$ $\square$

③ $10^8 \cdot 10^4 =$ $\square$

④ $2^{-3} =$ $\quad = \square$

⑤ $10^{-3} =$ $\quad = \square$

⑥ $\dfrac{5^3}{5} =$ $\quad = \square$

⑦ $\dfrac{9^7}{9^5} =$ $\quad = \square$

⑧ $\dfrac{9^5}{9^7} =$ $\quad = \square$

⑨ $(7^2)^3 =$ $\square$

⑩ $(-2)^{-3} =$ $\quad = \square$

Factoring out GCFs

(= Always attempt 1st!)

Name: ____________

Per: ____

FACTOR OUT THE COMMON MONOMIAL FROM EACH POLYNOMIAL.

ex] $2X+6 = \boxed{2\cdot(X+3)}$

1. $3X+6 =$ ___( )
2. $2X+12 =$ ___( )
3. $2X-12 =$ ___( )
4. $2X+2Y =$ ___( )
5. 3☆ + 3△ = ___( )
6. $6X+12 =$ ___( )
7. $2a+2b+2c =$ ___( )
8. $2a^2+4ab+2b^2 =$ ___( )
9. $X^2+2X =$ ___( )
10. $-12c^2+15c =$ ___( )
11. $10m^3n-20m^2n^2+5mn^3 =$ ___( )
12. $4x^5y-12x^4y^2+24x^3y^3 =$ ___( )

Adapted from Mark Burniston

FACTORING

Name: ______

Per: __

Def^n To Factor: To write as a multiplication problem

ex) 12

2 6

2 3

So, $12 = \boxed{2 \times 2 \times 3}$

ex) $2X+6$

$2 \quad (X+3)$

So, $2X+6 = \boxed{2 \cdot (X+3)}$

Factor. (Hint: Always factor out monomials first).

① $3X+12$
$= \boxed{\_\_(\quad)}$

② $12X-18$
$= \boxed{\_\_(\quad)}$

③ a^2-ab
$= \boxed{\_\_(\quad)}$

④ $6X^2-6X$
$= \boxed{\_\_(\quad)}$

⑤ $5y^2+25y$
$= \boxed{\_\_(\quad)}$

⑥ $21a^3-14a^2$
$= \boxed{\_\_(\quad)}$

⑦ $10a-35b+15$
$= \boxed{\_\_(\quad)}$

⑧ X^2+3X+2
$= \boxed{(\quad)(\quad)}$

⑨ $X^2+9X+20$
$= \boxed{(\quad)(\quad)}$

⑩ $X^2+6X-16$
$= \boxed{(\quad)(\quad)}$

⑪ $X^2-6X-16$
$= \boxed{(\quad)(\quad)}$

⑫ $X^2-10X+16$
$= \boxed{(\quad)(\quad)}$

⑬ $3X^2+X-2$
$= \boxed{(\quad)(\quad)}$

⑭ $8X^2+10X-3$
$= \boxed{(\quad)(\quad)}$

⑮ $8X^2+10X-12$
$= \boxed{(\quad)(\quad)}$

Solving Equations III
(by Factoring)

Directions: 1) First get one side of the equation to be ZERO.
2) Then factor, set each factor = 0, & solve each.

ex) $2X + 9 = \cancel{3}$
$\underline{-3 \quad -\cancel{3}}$
$2X + 6 = 0$
$= 2 \cdot (X+3) = 0$

So either $2 = 0$, or $X + \cancel{3} = 0$
$\underline{-\cancel{3} \quad -3}$
$2 \neq 0$; $\boxed{X = -3}$

ex) $X^2 + 5X + 6 = 0$
$= (X \; ?)(X \; ?) = 0$
$= (X \; 2)(X \; 3) = 0$
$= (X + 2)(X + 3) = 0$

So either $X + \cancel{2} = 0$ or $X + \cancel{3} = 0$
$\underline{-\cancel{2} \quad -2}$ $\quad$ $\underline{-\cancel{3} \quad -3}$
$\boxed{X = -2}$ or $\boxed{X = -3}$

Find X.

1. $(X+4)(X-5) = 0$
2. $0 = (X+1)(X+8)$
3. $X \cdot (X-6) = 0$
4. $X^2 + 8X + 15 = 0$
5. $X^2 - X - 12 = 0$
6. $X^2 + 10X + 16 =$
7. $X^2 + 8 = 9X$
8. $4x^2 - 36 = 0$
9. $3X^2 + 1 = 4X$

Name: ____________________

Per: ____

THE QUADRATIC FORMULA

☆ THE QUADRATIC FORMULA IS ONE OF THE MOST IMPORTANT FORMULAS IN ALGEBRA.

IT IS THE GENERAL SOLUTION OF ANY QUADRATIC EQUATION.

<u>THEOREM</u>: IF $ax^2 + bx + c = 0$ (& $a \neq 0$),

THEN $$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}.$$

(i.e. It finds the x-value for which a quadratic expression = zero) (and can be used when equal to other values too if rearranged)

NOTE: ALTHOUGH FACTORING CAN BE USED TO SOLVE SOME QUADRATIC EQUATIONS EQUAL TO ZERO (RELATIVELY QUICKLY), THE QUADRATIC FORMULA SOLVES <u>ALL</u> QUADRATIC EQUATIONS (ALTHOUGH USUALLY NOT AS QUICKLY).

<u>DIRECTIONS</u>: Use the Quadratic Formula to solve all of the following equations. (Be sure to always get 1 side equal to zero 1st)

1. $x^2 - 8x + 15 = 0.$

$x = \boxed{\quad \& \quad}$

2. $x^2 = 10x.$

$x = \boxed{\quad \& \quad}$

Adapted from Mark Burniston

~~3. $x^2+6x-7=0$.~~

$x = \boxed{\quad \text{\&} \quad}$

~~4. $x^2+54=-15x$~~

$x = \boxed{\quad \text{\&} \quad}$

5. $2x^2+15x-8=0$.

$x = \boxed{\quad \text{\&} \quad}$

6. $10x^2+43x+28=0$.

$x = \boxed{\quad \text{\&} \quad}$

7. $x^2-4x+1=0$.

$x = \boxed{\quad \text{\&} \quad}$

8. $x^2-x-1=0$.

$x = \boxed{\quad \text{\&} \quad}$

9. $x^2-6x+25=0$.

$x = \boxed{\quad \text{\&} \quad}$

10. $9x^2+11=12x$.

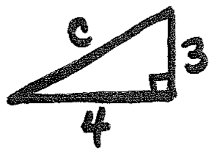
$x = \boxed{\quad \text{\&} \quad}$

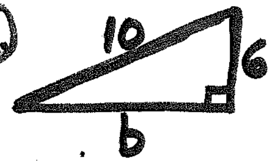
Name: ______________

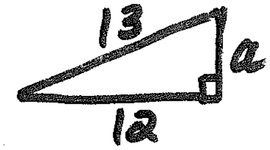
GEOMETRY

Per: __

A. LENGTH ($a^2+b^2=c^2$)

(1) 

(2) 

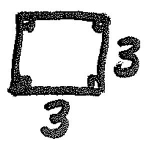
(3) 

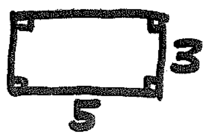
c =

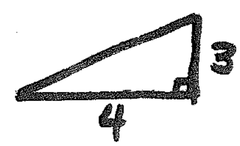
b =

a =

B. PERIMETER & CIRCUMFERENCE ($C = \pi d = 2\pi r$)

(4) 

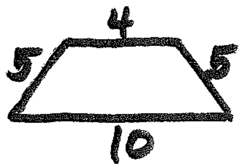
(5) 

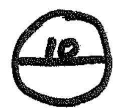
(6) 

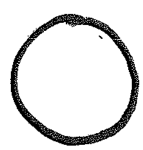
P =

P =

P =

(7) 

(8) 

(9) 

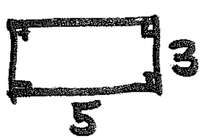
$C = 16\pi$

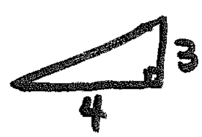
P =

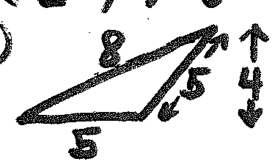
C =

r =

C. AREA ($A_{\square} = bh$, $A_{\triangle} = \frac{1}{2}bh$, $A_{\text{trap}} = \overbrace{\left(\frac{b_1+b_2}{2}\right)}^{\text{(=average base)}}h$, $A_{\circ} = \pi r^2$)

(10) 

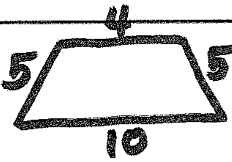
(11) 

(12) 

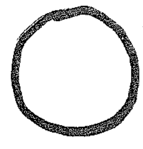
A =

A =

A =

(13) 

(14) 

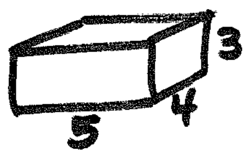
(15) 
$C = 18\pi$

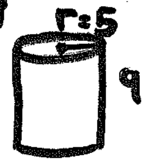
A=

A=

A=

D. SURFACE AREA ($SA = 2\pi rh + 2\pi r^2$, $SA = 4\pi r^2$)

(16) 

(17) 

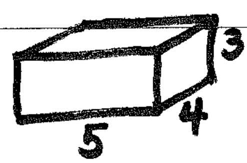
(18) 
r=3

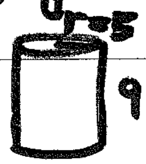
SA=

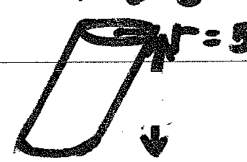
SA=

SA=

E. VOLUME ($V = LWH$, $V = \pi r^2 h$, $V = \frac{1}{3}Bh$, $V = \frac{4}{3}\pi r^3$)

(19) 

(20) 

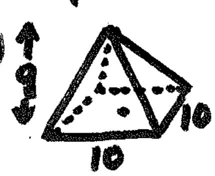
(21) 

V=

V=

V=

(22) 

(23) 

(24) 
r=3

V=

V=

V=

Name: ____________

INVERSE FUNCTIONS

Per: ____

Def^{n} INVERSE FUNCTION, $f^{-1}(x)$: A FUNCTION THAT REVERSES THE EFFECT OF A GIVEN FUNCTION $f(x)$ (TAKES "y" BACK TO "x").

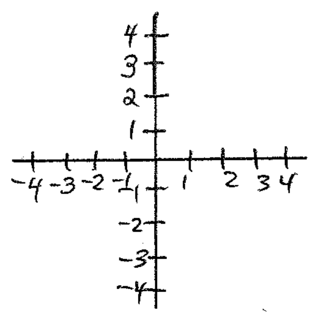
* NOTE: INVERSE FUNCTIONS ONLY EXIST FOR FUNCTIONS WHERE y-VALUES ONLY OCCUR ONCE.
(ie. for strictly increasing or strictly decreasing functions

STEPS:

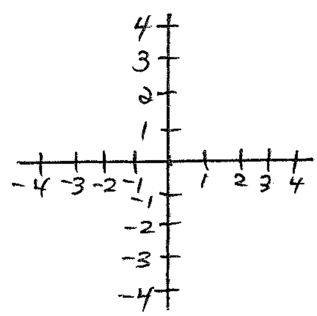
1) Solve for x.
2) Switch x & y.
3) Re-label y as y^{-1}.

Find and graph the inverse function y^{-1} for the given function y.

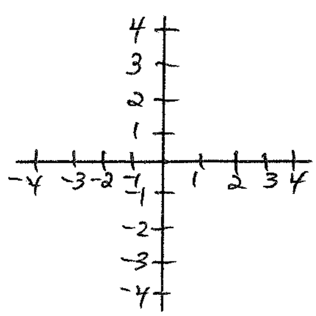
① $y = x + 3$.



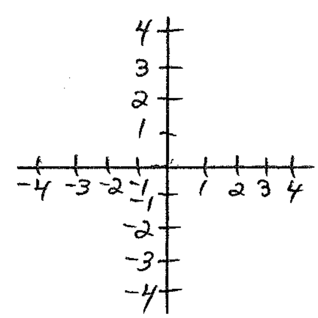
② $y = 2x$.

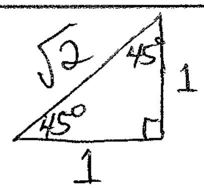


③ $y = 2x + 1$.

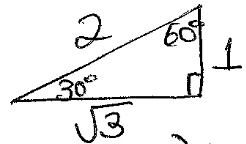


④ $y = x^2,\ x \geq 0$.





TRIGONOMETRIC FUNCTIONS



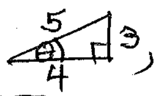
RECALL THE DEFINITIONS OF THE 6 TRIGONOMETRIC (CIRCULAR) FUNCTIONS:

=fractions

$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$,

$\csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite}}$, $\sec(\theta) = \frac{\text{hypotenuse}}{\text{adjacent}}$, $\cot(\theta) = \frac{\text{adjacent}}{\text{opposite}}$

(*Remember "SOH CAH TOA" for the first three above)

1. Given  find:

$\sin\theta =$ ☐ $\csc\theta =$ ☐
$\cos\theta =$ ☐ $\sec\theta =$ ☐
$\tan\theta =$ ☐ $\cot\theta =$ ☐

2. Find:

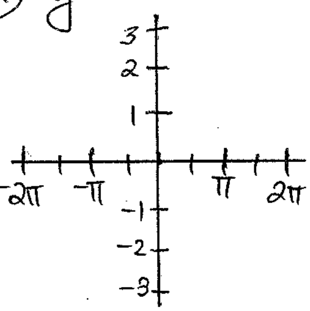
$\sin 45° =$ ☐
$\cos 45° =$ ☐
$\tan 45° =$ ☐

3. Find:

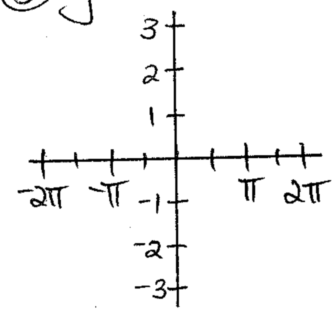
$\sin 30° =$ ☐ $\sin 60° =$ ☐
$\cos 30° =$ ☐ $\cos 60° =$ ☐
$\tan 30° =$ ☐ $\tan 60° =$ ☐

Graph the following functions for various angles.

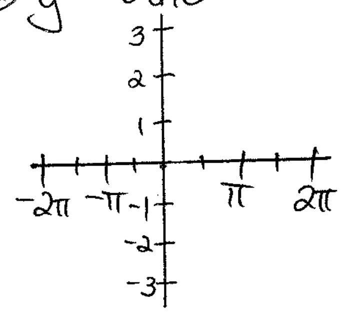
4. $y = \sin\theta$



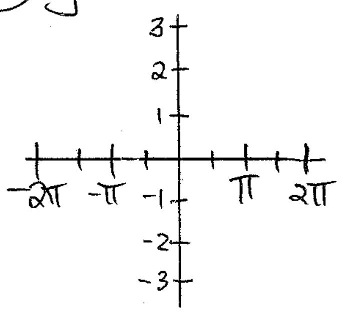
5. $y = \cos\theta$



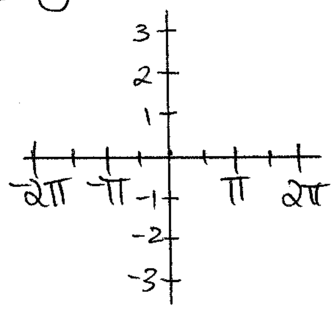
6. $y = \tan\theta$



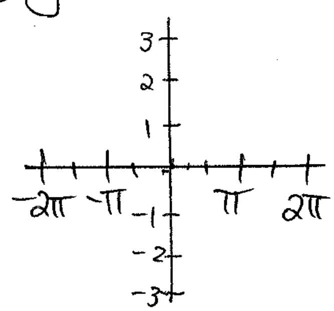
7. $y = \csc\theta$



8. $y = \sec\theta$



9. $y = \cot\theta$



Name: ____________

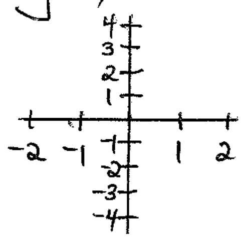
Graphing Common Functions

Per: __

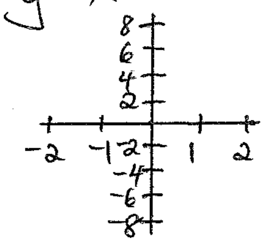
Use an xy Table to graph the following functions.

A. Polynomials

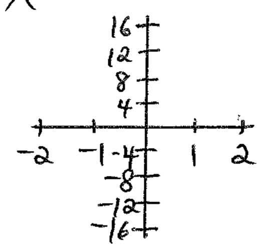
1. $y = x^2$



2. $y = x^3$

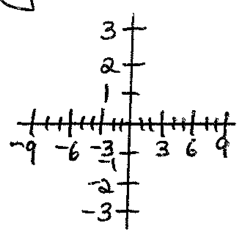


3. $y = x^4$

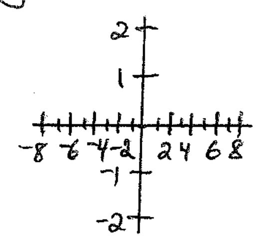


B. Radicals

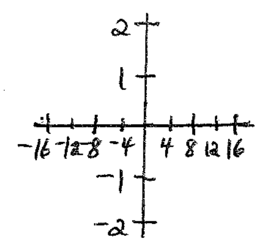
4. $y = x^{\frac{1}{2}}$ $(= \sqrt{x})$



5. $y = x^{\frac{1}{3}}$ $(= \sqrt[3]{x})$

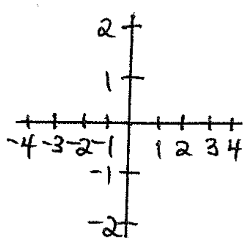


6. $y = x^{\frac{1}{4}}$ $(= \sqrt[4]{x})$

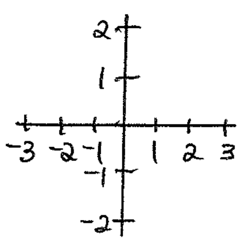


C. Rationals

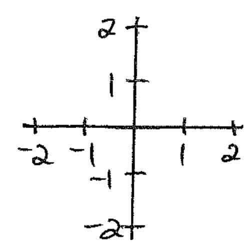
7. $y = \frac{1}{x}$



8. $y = \frac{1}{x^2}$



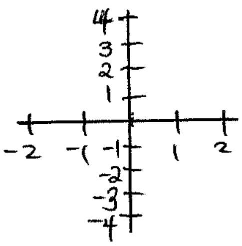
9. $y = \frac{1}{x^4}$

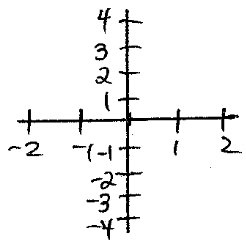


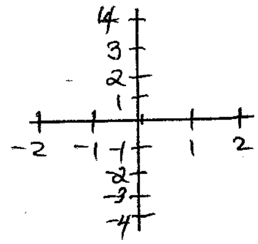
Graphing TRANSLATED FUNCTIONS

Use an xy Table to accurately graph the following translated functions. (Then try to notice the effect.)

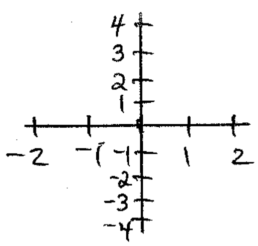
1. $y = x^2$ 2. $y = x^2 + 1$ 3. $y = x^2 - 1$

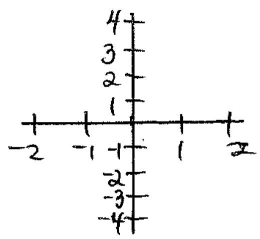


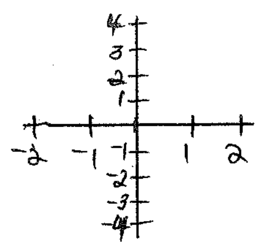




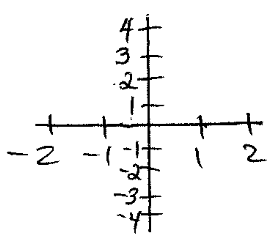
4. $y = 2x^2$ 5. $y = \frac{1}{2}x^2$ 6. $y = -x^2$

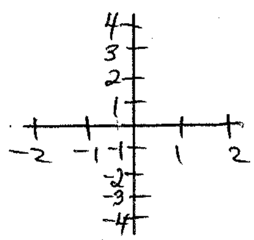


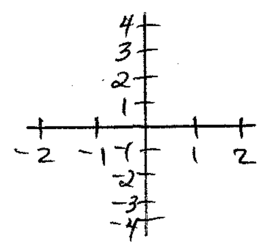




7. $y = (x+1)^2$ 8. $y = (x-1)^2$ 9. $y = -\frac{1}{2}(x+1)^2 + 2$







Name: ______________________

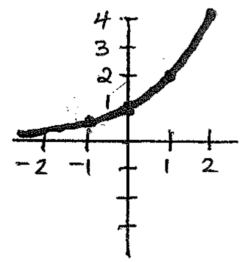
Per: ____

EXPONENTIALS

Def'n <u>EXPONENTIAL FUNCTION</u>: A FUNCTION WITH A CONSTANT BASE $b > 0$ AND A VARIABLE EXPONENT.

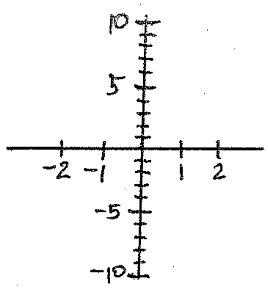
<u>ex</u> Graph $y = 2^x$.

x	$y = 2^x$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$



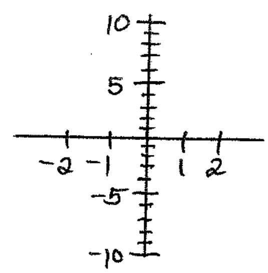
(1.) Graph $y = 3^x$.

x	$y = 3^x$
-1	
0	
1	
2	



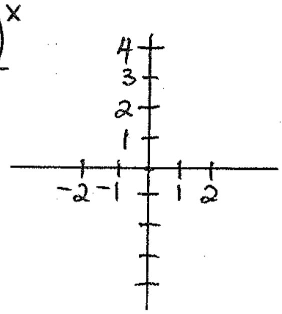
(2) Graph $y = 10^x$

x	$y = 10^x$
-2	
-1	
0	
1	



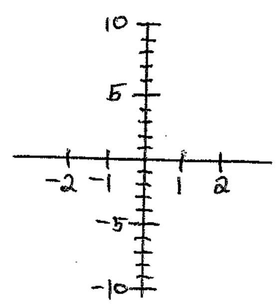
(3.) Graph $y = \left(\frac{1}{2}\right)^x$.

x	$y = \left(\frac{1}{2}\right)^x$
-2	
-1	
0	
1	



(4.) Graph $y = e^x$. $(e \approx 2.71)$

x	$y = e^x$
-1	
0	
1	
2	



Adapted from Mark Burniston

LOGARITHMS

Def^n LOGARITHMIC FUNCTION: A FUNCTION WITH A CONSTANT BASE $b > 0$ AND A VARIABLE POWER FOR WHICH THE EXPONENT NEEDED ON b TO ARRIVE AT THE POWER IS SOUGHT.

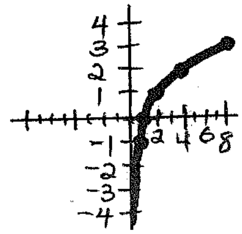
i.e. $\boxed{\log_b y = x \iff y = b^x}$ ← (exponent = sought)

("power") ↗ y $\boxed{3}$ ← (exponent needed)

ex| $\log_2 8 = \boxed{3}$ because $8 = 2^{\boxed{3}}$.

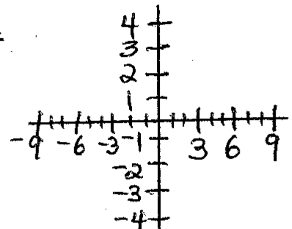
ex| Graph $y = \log_2 x$.

x	$y = \log_2 x$
$\frac{1}{2}$	$\log_2(\frac{1}{2}) = -1$
1	$\log_2(1) = 0$
2	$\log_2(2) = 1$
4	$\log_2(4) = 2$
8	$\log_2(8) = 3$



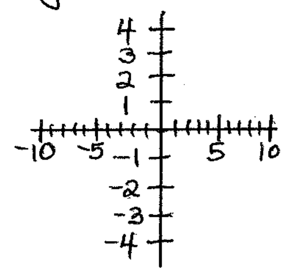
① Graph $y = \log_3 x$.

x	$y = \log_3 x$
$\frac{1}{3}$	
1	
3	
9	



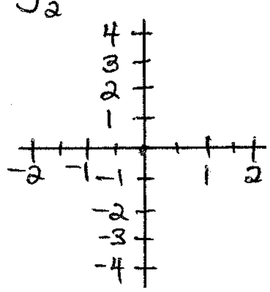
② Graph $y = \log x$. ($\log = \log_{10}$)

x	$\log x$
$\frac{1}{100}$	
$\frac{1}{10}$	
1	
10	



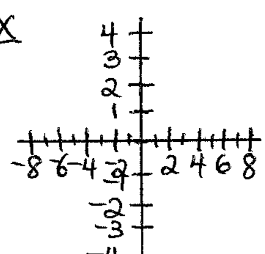
③ Graph $y = \log_{\frac{1}{2}} x$.

x	$\log_{\frac{1}{2}} x$
$\frac{1}{4}$	
$\frac{1}{2}$	
1	
2	



④ Graph $y = \ln(x)$. ($\ln = \log_e$)

x	$y = \ln x$
$\frac{1}{e}$	
1	
e	
e^2	



Name: ____________________

Per: ____

LIMITS I
(→infinite proximity)
(BASIC)

A] NUMERICAL TABLE & GRAPHING

Use a numerical table for #s 1–3 & graph #s 4–10 (÷ #10 1st).

① $\lim_{x \to 5} (2x+1) = \boxed{}$

x:	4.9	4.99	5.01	5.1
$2x+1$:				

② $\lim_{x \to 3} (x^2) = \boxed{}$

x:	2.9	2.99	3.01	3.1
x^2:				

③ $\lim_{x \to 0} (|x|^x) = \boxed{}$

x:	−.1	−.01	.01	.1
$|x|^x$:				

④ $\lim_{x \to 0} \left(\frac{1}{x}\right) = \boxed{}$

⑤ $\lim_{x \to 0} \left(\frac{1}{x^2}\right) = \boxed{}$

⑥ $\lim_{x \to \infty} (e^x) = \boxed{}$

⑦ $\lim_{x \to \infty} (e^{-x}) = \boxed{}$

⑧ $\lim_{x \to \infty} (\tan^{-1} x) = \boxed{}$

⑨ $\lim_{x \to -\infty} (\tan^{-1} x) = \boxed{}$

⑩ $\lim_{x \to \infty} \frac{2x-1}{x-1} = \boxed{}$

B] DIRECT SUBSTITUTION

Frequently limits are the as the y-value <u>at</u> the x-value (so always try direct substitution first).

① $\lim_{x \to 5} (2x+1) = \boxed{}$

② $\lim_{x \to 3} (x^2) = \boxed{}$

③ $\lim_{x \to 2} \left(\frac{1}{x}\right) = \boxed{}$

④ $\lim_{x \to 4} (\sqrt{x}) = \boxed{}$

⑤ $\lim_{x \to 5} \left(\frac{|x|}{x}\right) = \boxed{}$

⑥ $\lim_{x \to -3} \left(\frac{|x|}{x}\right) = \boxed{}$

LIMITS II
(ALGEBRAIC, & @ INFINITY)

C. ALGEBRAIC TECHNIQUES

Although $\frac{\text{Non-zero}}{0}$ = undefined, as the denominator → 0, the fraction → ∞. But $\frac{0}{0}$ = Indeterminant and requires techniques such as factor & reduce and common denominator.

1. $\lim_{x\to 6} \frac{x^2-36}{x-6} =$ $= \boxed{}$
2. $\lim_{x\to 6} \frac{x-6}{x^2-36} =$ $= \boxed{}$
3. $\lim_{x\to 9} \frac{2x-18}{5x-45} =$ $= \boxed{}$
4. $\lim_{x\to 1} \frac{x^2-3x+2}{x-1} =$ $= \boxed{}$
5. $\lim_{x\to 2} \frac{3x^2-4x+4}{2x^2-8} =$ $= \boxed{}$
6. $\lim_{x\to 4} \frac{x^2-5x+6}{x} =$ $= \boxed{}$
7. $\lim_{x\to 0} \frac{x^3}{x} =$ $= \boxed{}$
8. $\lim_{x\to 0} \frac{|x|}{x} =$ $= \boxed{}$
9. $\lim_{x\to 1} \left[\frac{1}{1-x} - \frac{2}{1-x^2}\right] =$ $= \boxed{}$
10. $\lim_{x\to 4} \left[\frac{1}{\sqrt{x}-2} - \frac{4}{x-4}\right] =$ $= \boxed{}$

D. AT INFINITY

Two basic limits at infinity are $\lim_{x\to\infty}(x) = \infty$ and $\lim_{x\to\infty}\left(\frac{1}{x}\right) = 0$. $\frac{\infty}{\infty}$ = Indeterminant & requires dividing numerator & denominator by the largest power of x in the denominator.

1. $\lim_{x\to -\infty}(x) =$ $\boxed{}$
2. $\lim_{x\to -\infty}\left(\frac{1}{x}\right) =$ $\boxed{}$
3. $\lim_{x\to \infty}(x^2) =$ $\boxed{}$
4. $\lim_{x\to -\infty}(x^2) =$ $\boxed{}$
5. $\lim_{x\to \infty}(2x+1) =$ $\boxed{}$
6. $\lim_{x\to \infty}(\sqrt{x}) =$ $\boxed{}$
7. $\lim_{x\to \infty}\left(\frac{4}{x+5}\right) =$ $\boxed{}$
8. $\lim_{x\to \infty}\left(\frac{7x}{8x+9}\right) =$ $\boxed{}$
9. $\lim_{x\to \infty}\left(\frac{3x^2+2}{4x+5}\right) =$ $\boxed{}$
10. $\lim_{x\to \infty}\left(\frac{3x+2}{4x^2+5}\right) =$ $\boxed{}$

(THIS IS AN IMPORTANT CALCULUS-READINESS WORKSHEET.) Name:

RATIOS & RATES

Per:

DEFINITIONS

1) RATIO: THE RELATIONSHIP BETWEEN 2 (OR MORE) QUANTITIES. (ex 2 parts orange juice to 1 part pineapple juice (2:1))
2) FRACTION: THE RATIO OF PARTS TO THE WHOLE (ex $\frac{1}{3}$); INDICATES THE MULTIPLICATIVE RELATIONSHIP BETWEEN 2 QUANTITIES.
3) RATE: A (2-ENTITY) RATIO HAVING DIFFERENT UNITS OF MEASURE. (ex $\frac{\text{miles}}{\text{hour}}$)
4) UNIT RATE: A RATE WITH DENOMINATOR EQUAL TO 1 (FOUND BY DIVIDING).

FIND THE FOLLOWING UNIT RATES. (YOU MAY USE A CALCULATOR)

1. 6 taco sauces for 2 tacos = $\boxed{\frac{\text{taco sauces}}{1 \text{ taco}}}$
2. 2 burgers for \$5 = $\boxed{\frac{\$}{1 \text{ burger}}}$
3. 60 minutes for 40 questions = $\boxed{\frac{\text{minutes}}{1 \text{ question}}}$
4. 100 miles in 2 hours = $\boxed{\frac{\text{miles}}{1 \text{ hour}}}$
5. \$18 trillion of debt for 300 million Americans = $\boxed{\frac{\$}{1 \text{ American}}}$
6. \$18 trillion of gross domestic product for 300 million Americans = $\boxed{\frac{\$}{1 \text{ American}}}$
7. 700,000 Alaskans in 600,000 square miles = $\boxed{\frac{\text{Alaskans}}{1 \text{ mi}^2}}$
8. A 100-pound woman standing on one $\frac{1}{4}$-in^2 high heel = $\boxed{\frac{\text{pounds}}{1 \text{ in}^2}}$
9. 2.4 million births compared to 300 million Americans = $\boxed{\frac{\text{\% births}}{1 \text{ American}}}$
10. 3 million global deaths due to hunger per year = $\boxed{\frac{\text{deaths}}{1 \text{ minute}}}$
11. 500,000 U.S. murders per year = $\boxed{\frac{\text{murders}}{1 \text{ minute}}}$
12. 400 miles driven using 16 gallons of gasoline = $\boxed{\frac{\text{miles}}{1 \text{ gallon}}}$

~~(THIS IS AN IMPORTANT~~ CALCULUS-READINESS WORKSHEET.)

RATE PROBLEMS

★ RECALL THE FORMULA $\boxed{\text{RATE} \times \text{TIME} = \text{DISTANCE}}$ ($R \cdot T = D$)

(OR MORE GENERALLY RATE × DENOMINATOR QUANTITY = NUMERATOR QUANTITY).

→ ONCE YOU KNOW A (UNIT) RATE, YOU CAN FIND OTHER DESIRED QUANTITIES (USUALLY) BY MULTIPLYING.

CALCULATE THE FOLLOWING. (YOU MAY USE A CALCULATOR.)

1. How many seconds are in 1 year?
2. If your heart rate is 80 beats per minute, how many times does it beat in a day?
3. If mercury weighs .88 pounds per ounce, how much will one cup of mercury weigh? (1 cup = 8 oz)
4. If your computer can download files at a rate of 20 M bits per second, how long will it take to download a 100 Mbit file?
5. If the inflation rate is 3% per year, how much will a $50 item cost 1 year from now?
6. If you make $20 per 1 hour, work 8 hours per day, 5 days per week, & 50 weeks per year, how much will you make in 1 year?
7. If you read at a rate of 200 words per minute, and there are 300 words per page, how long will it take to read 400 pages?
8. If a printer can print at a rate of 8 pages per minute, how long will it take to print 50 pages?
9. If it takes 10 hours to paint 1 apartment, how many 8-hour days will it take to paint 50 apartments?
10. If a tank is leaking at a rate of 10 gallons per minute, how many gallons will be leaked after 1 day?
11. If you walk at a rate of 3 miles per hour, how far will you walk in $1\frac{1}{2}$ hours?
12. If the speed of light is 186,000 miles per second, how many times will it go around the Earth in 1 second? (Earth's circumference ≈ 25,000 miles)

Adapted from Mark Burniston

(BASIS FOR 1ST HALF OF CALCULUS!) Name: ______________________

Per: ___

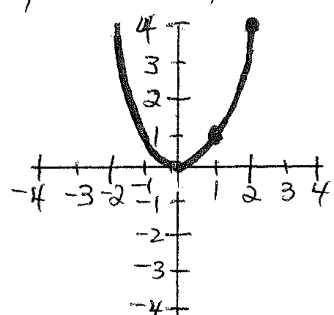
APPROXIMATING SLOPES

GIVEN 2 POINTS (x_1, y_1) & (x_2, y_2), THE SLOPE OF THE LINE BETWEEN THEM IS $\boxed{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$.

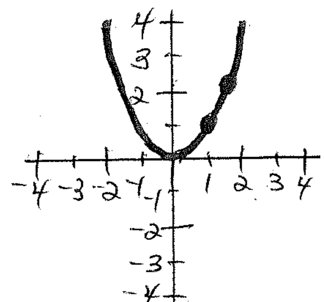
THE CLOSER THE 2 POINTS ARE, THE BETTER OUR APPROXIMATION OF THE SLOPE AT ONE POINT WILL BE.

DIRECTIONS: For each problem, approximate the slope of $y = x^2$ at the first point by calculating and graphing the slope of the line through it and the second point.

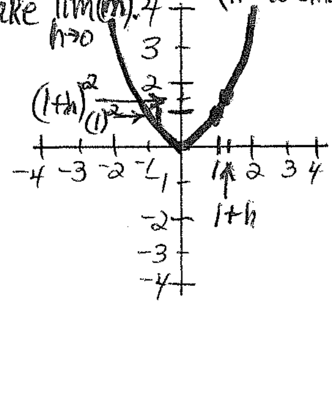
1. At $(1,1)$ from $(2,4)$.



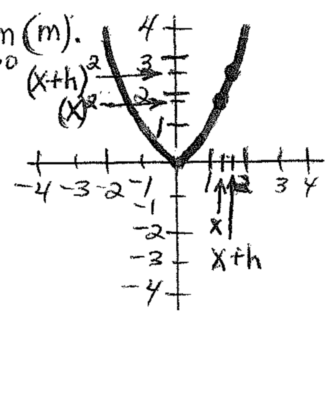
2. At $(1,1)$ from $(1\frac{1}{2}, 2\frac{1}{4})$.



3. At $(1,1)$ from $(1+h, (1+h)^2)$. (h = a small #)
Then take $\lim_{h \to 0} (m)$.



4. At (x, x^2) from $(x+h, (x+h)^2)$.
Then take $\lim_{h \to 0} (m)$.



(= BASIS FOR 1ST HALF OF CALCULUS (cont'd)!)

DERIVATIVES OF POWER FUNCTIONS

Def^n: DERIVATIVE: THE LIMIT OF THE RATE OF CHANGE OF A DEPENDENT VARIABLE TO ITS INDEPENDENT VARIABLE AT A POINT X AS THE CHANGE IN THE INDEPENDENT VARIABLE $\to 0$.
(i.e. LIMIT OF SLOPE OR RATIO OF INFINITESIMAL CHANGES)

$$\boxed{f'(x) \overset{def^n}{=} \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}} \left(= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}\right).$$

(i.e. = Ratio of infinitesimal changes) ($\Delta y = y_2 - y_1$, $\Delta x = x_2 - x_1$)

FOR POWER FUNCTIONS, THE DERIVATIVE CALCULATION SIMPLIFIES TO

$$\boxed{(x^n)' = nx^{n-1}}$$ (= SHORTCUT RESULT)

DIRECTIONS: Find y' and evaluate it at the indicated x-values.

1. $y = 2x + 1$.
   a) $y'(x) =$
   b) $y'(0) =$
   c) $y'(1) =$
   d) $y'(2) =$
   e) $y'(-1) =$

2. $y = x^2$.
   a) $y'(x) =$
   b) $y'(0) =$
   c) $y'(1) =$
   d) $y'(2) =$
   e) $y'(-1) =$

3. $y = x^2 + 1$. ($c' = 0$)
   a) $y'(x) =$
   b) $y'(0) =$
   c) $y'(1) =$
   d) $y'(2) =$
   e) $y'(0) =$

4. $y = x^3$.
   a) $y'(x) =$
   b) $y'(0) =$
   c) $y'(1) =$
   d) $y'(2) =$
   e) $y'(-1) =$

5. $y = \frac{1}{x} =$ $\quad$.
   a) $y'(x) =$ $\quad =$
   b) $y'(0) =$
   c) $y'(1) =$
   d) $y'(2) =$
   e) $y'(-1) =$

6. $y = \sqrt{x} =$ $\quad$.
   a) $y'(x) =$ $\quad =$
   b) $y'(0) =$
   c) $y'(1) =$
   d) $y'(4) =$
   e) $y'(-1) =$

7. $y = \sqrt[3]{x} =$ $\quad$.
   a) $y'(x) =$ $\quad =$
   b) $y'(0) =$
   c) $y'(1) =$
   d) $y'(8) =$
   e) $y'(-1) =$

8. $y = x^2 + 2x + 1$.
   a) $y'(x) =$
   b) $y'(0) =$
   c) $y'(1) =$
   d) $y'(2) =$
   e) $y'(-1) =$

9. $y = (x-2)^2 + 1$
   $=$ $\quad$.
   a) $y'(x) =$
   b) $y'(0) =$
   c) $y'(1) =$
   d) $y'(2) =$
   e) $y'(3) =$

~~(OPTIONAL, BUT – BASIS FOR 2^{ND} HALF OF CALCULUS!)~~ ~~FOR AB~~ Name: ______

APPROXIMATING AREAS

Per: ____

RECALL THE AREA OF A RECTANGLE IS $\boxed{A_{\square} = bh}$.

WE CAN APPROXIMATE THE AREA UNDER A CURVE (ABOVE THE X-AXIS) BY DRAWING RECTANGLES UNDER THE CURVE AND ADDING UP ALL OF THEIR INDIVIDUAL AREAS.

FURTHERMORE, THE MORE RECTANGLES WE DRAW, THE BETTER OUR APPROXIMATION TO THE ACTUAL AREA WILL BE.

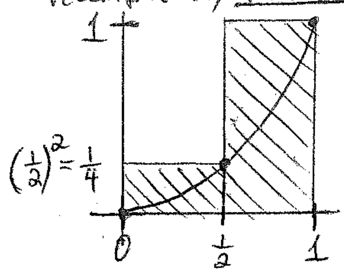
DIRECTIONS: Approximate the area under $y = x^2$ from $x=0$ to $x=1$ by adding the areas of the (indicated number of) rectangles ("n") (Use their Right sides for heights)

1. $n = 2$.

$A \approx A_1 + A_2$

$\approx$

$\approx \boxed{}$

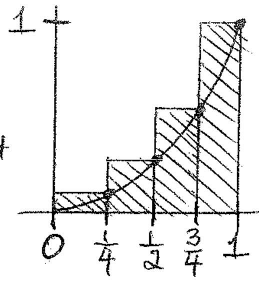


2. $n = 4$.

$A \approx A_1 + A_2 + A_3 + A_4$

$\approx$

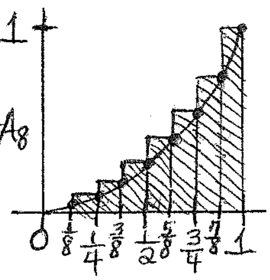
$\approx \boxed{}$



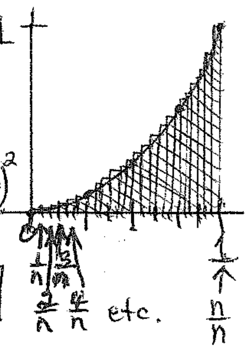
3. $n = 8$.

$A \approx A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8$

$\approx$



4. $n = n$.



$A \approx A_1 + A_2 + A_3 + \cdots + A_n$

$= \left(\frac{1}{n}\right)\left(\frac{1}{n}\right)^2 + \left(\frac{1}{n}\right)\left(\frac{2}{n}\right)^2 + \cdots + \left(\frac{1}{n}\right)\left(\frac{n}{n}\right)^2$

$= \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \cdots + \left(\frac{n}{n}\right)^2\right]$

$= \frac{1}{n}\left(\frac{1}{n^2}\right)\left[(1)^2 + (2)^2 + \cdots + (n)^2\right]$

(Formula) $= \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$

$= \frac{1}{n^3}\left[\frac{2n^3 + 3n^2 + n}{6}\right] = \frac{1}{6}\left[2 + \frac{3}{n} + \frac{1}{n^2}\right]$.

$= \boxed{}$.

And $\lim_{n \to \infty} \left\{\frac{1}{6}\left[2 + \cancel{\frac{3}{n}}^{\,0} + \cancel{\frac{1}{n^2}}^{\,0}\right]\right\} = \frac{2}{6} = \boxed{\frac{1}{3} = A}$.

Adapted from Mark Burniston

(OPTIONAL, BUT = BASIS FOR 2^{nd} HALF OF CALCULUS (cont'd)!) FOR AB

ANTI-DERIVATIVES OF POWER FUNCTIONS

Def^n ANTI-DERIVATIVE (F): A FUNCTION WHOSE DERIVATIVE IS THE ORIGINAL FUNCTION (f) GIVEN.

★ BECAUSE ANTI-DIFFERENTIATION IS THE REVERSE PROCESS OF DIFFERENTIATION, THE ORIGINAL FUNCTION CAN BE THOUGHT OF AS A RATE THAT IS BEING MULTIPLIED BY ITS DENOMINATORIAL QUANTITY, THUS RESULTING IN (THE CHANGE IN) THE NUMERATORIAL QUANTITY (= THE ANTI-DERIVATIVE FUNCTION).

IN OTHER WORDS, THE ANTI-DERIVATIVE FUNCTION IS THE PRODUCT-SUM FUNCTION FOR A GIVEN ORIGINAL FUNCTION. (i.e. THE LIMIT OF THE SUM OF AN INFINITE NUMBER OF PRODUCTS.)

$$\boxed{F(x) = \int_0^x f(t)\,dt = \lim_{n\to\infty} \sum_{i=0}^{x_n} f(t_i)\Delta t} \quad (\& \Rightarrow F'(x) = f(x))$$

FOR POWER FUNCTIONS, THE ANTI-DERIVATIVE CALCULATION SIMPLIFIES TO

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + C}$$

(= SHORTCUT RESULT) ("C" = arbitrary constant) (& = AREA UNDER x^n FROM 0 TO "x".)

(& = REVERSE OF POWER RULE FOR DERIVATIVES.)

DIRECTIONS: Find $F(x) = \int f(x)dx$ for the given function $f(x)$ and evaluate it at the indicated x-values.

1. $f(x) = 2 \; (= 2x^0)$.
   a) $F(x) =$
   b) $F(0) =$
   c) $F(1) =$
   d) $F(2) =$

2. $f(x) = 2x$.
   a) $F(x) =$
   b) $F(0) =$
   c) $F(1) =$
   d) $F(2) =$

3. $f(x) = x^2$.
   a) $F(x) =$
   b) $F(0) =$
   c) $F(1) =$
   d) $F(2) =$
   e) $F(2) - F(1) =$

4. $f(x) = x + x^{\frac{1}{2}} + 1$.
   a) $F(x) =$
   b) $F(0) =$
   c) $F(1) =$
   d) $F(4) =$

(BC only)

~~CALCULUS AB~~ FINAL REVIEW (PART 0): FUNDAMENTALS

Name: ______

Per: ______

FIND THE FOLLOWING.

1. $\lim_{x \to -3} \frac{x+3}{x^2+9} =$

2. $(x^3+2x^2+5x+6)' =$

3. At which x-value(s) could $y = 2x^3 - 3x^2 - 36x + 54$ have a local maximum or local minimum?

4. $\int (x^2+4x+5)\,dx =$

(BC only)

~~CALCULUS AB~~ Name:

FINAL REVIEW (PART I):
BASIC PROBLEMS

Per: ___

FIND THE FOLLOWING.

(1.) $\lim\limits_{x \to 2} \dfrac{x^2+3x-10}{x-2} =$

(2.) a) A PHYSICAL CHARACTERISTIC OF THE GRAPHS OF DIFFERENTIABLE FUNCTIONS IS THAT THEY ARE ________

b) TRUE OR FALSE:

(i) IF A FUNCTION IS CONTINUOUS, THEN IT IS DIFFERENTIABLE.

(ii) IF A FUNCTION IS DIFFERENTIABLE, THEN IT IS CONTINUOUS.

(3.) a) $(\sin x)' =$ b) $(\cos x)' =$ c) $(\tan x)' =$

d) $(\csc x)' =$ e) $(\sec x)' =$ f) $(\cot x)' =$

g) $(e^x)' =$ h) $(xe^x)' =$ i) $(e^{2x})' =$

(4.) a) $\int e^x dx =$ b) $\int \sin x\,dx =$

c) $\int \frac{1}{x} dx =$ d) $\int \frac{1}{x-1} dx =$

(5.) a) $\int 3x^2 \sin(x^3)\,dx =$

b) THE AREA UNDER $y = x^2$ FROM $x=0$ TO $x=2$ IS

c) THE AREA BETWEEN $y=x^2$ & $y=x$ IS

(6) SOLVE $\frac{dy}{dx} = x^2 y$.

(BC only)

CALCULUS AB
FINAL REVIEW (PART II):
COMMON PROBLEMS

Name:

Per:

FIND THE FOLLOWING.

1. $\lim_{x \to 0} \frac{e^{5x}}{x^2} =$

2. a) $(b^x)' =$ b) $(\ln x)' =$
   c) $(\log_b x)' =$ d) $y = (\sin^{-1} x)' =$ (Solve for x to prove.)

3. FIND THE
   a) ~~FORMULA~~ FOR THE SLOPES OF ALL TANGENT LINES TO $y = x^2$.
   b) SLOPE OF THE TANGENT LINE TO $y = x^2$ AT $x = 2$.
   c) THE EQUATION OF THE TANGENT LINE TO $y = x^2$ AT $x = 2$.

4. a) WHAT IS THE DEFINITION OF A CRITICAL POINT?
   b) FOR $y = x^3 + 3x^2 - 9x$,
      (i) FIND WHERE IT HAS A LOCAL MINIMUM &/OR MAXIMUM (BY FINDING WHERE THE SLOPE OF ITS TANGENT LINES EQUALS 0) (OR DOES NOT EXIST)

      (ii) ITS ABSOLUTE MAXIMUM & MINIMUM ON $x \in [-10, 10]$.
   c) SKETCH ITS GRAPH.

→

(5.) a) $\int b^x\,dx$ b) $\int \frac{1}{2x-1}\,dx$

c) FIND $x(t)$ = THE POSITION OF AN OBJECT, GIVEN
(i) ITS CONSTANT ACCELERATION IS $a = 32$,
(ii) ITS INITIAL VELOCITY IS $V(0) = 0$, &
(iii) ITS INITIAL POSITION IS $X(0) = 0$.

d) FIND THE AVERAGE HEIGHT OF $y = x^2$ ON THE INTERVAL FROM $X=0$ TO $X=2$.

e) USE THE DISK METHOD TO FIND THE VOLUME OF THE SOLID OBTAINED BY ROTATING THE CURVE $y = \sqrt{x}$ ABOUT THE X-AXIS FROM $X=0$ TO $X=3$.

(6) SKETCH THE SLOPE FIELD FOR $y' = x$.

(BC only) ~~Calculus AB~~ Name:

Per: ___

Final Review (Part III): Less Common Problems

Find the following.

1. a) Find y' for $x^2+xy-y^2=5$.

b) Find y' for $y=\frac{(x+1)^2(2x^2-3)}{x^2+1}$.

c) Approximate $\sqrt{10}$ by approximating the function $y=\sqrt{x}$ with its tangent line at $x=9$.

d) If $y=x^2$, find $(y^{-1})'(x)$ at $x=9$.

2. If a rectangular box of length $L=12$ in. & width $W=6$ in. is being filled with water at a rate of 18 in^3/min, how fast is the height of the water rising?

(3.) APPROXIMATE THE AREA UNDER $y=x^2$ FROM $x=0$ TO $x=1$ WITH 4 RIGHT-ENDPOINT RECTANGLES.

(4.) USE THE CYLINDRICAL SHELL METHOD TO FIND THE VOLUME OF THE SOLID OBTAINED BY ROTATING THE REGION ABOVE $y=x^2+1$ ABOUT THE y-AXIS FROM $x=0$ TO $x=3$.

Adapted from Mark Burniston