AP Physics C: Prep
Homework for Day 8

Attached to this document are scans of 3 pages from a calculus textbook (Calculus, 5th edition, late trigonometry version by Swokowski (1992)). Please do the following problems on a separate sheet of paper. On these pages, be sure to include not only the problem number, but also the section that it comes from. All of these problems are applications of basic integral properties and identities.

Homework for Session 1:

- Exercises 5.1, problems 1 – 14 odd
    - Note that for problems 11 – 14, you have to FOIL out the function. You cannot use the chain rule on this, because the chain rule only works for derivatives. In the next unit, you will learn the integral equivalent of the chain rule: u-substitution.
- Exercises 8.4, problems 1 – 4 odd

Homework for Session 2:

- Exercises 5.1, problems 1 – 14 even
    - Note that for problems 11 – 14, you have to FOIL out the function. You cannot use the chain rule on this, because the chain rule only works for derivatives. In the next unit, you will learn the integral equivalent of the chain rule: u-substitution.
- Exercises 8.4, problems 1 – 4 even

Swokowski 1992
Calculus, 5th edition (late trigonometry version)

EXERCISES 5.1

Exer. 1–24: Evaluate.

1 $\int (4x + 3)\,dx$

2 $\int (4x^2 - 8x + 1)\,dx$

3 $\int (9t^2 - 4t + 3)\,dt$

4 $\int (2t^3 - t^2 + 3t - 7)\,dt$

5 $\int \left(\frac{1}{z^3} - \frac{3}{z^2}\right) dz$

6 $\int \left(\frac{4}{z^7} - \frac{7}{z^4} + z\right) dz$

7 $\int \left(3\sqrt{u} + \frac{1}{\sqrt{u}}\right) du$

8 $\int (\sqrt{u^3} - \frac{1}{2}u^{-2} + 5)\,du$

9 $\int (2v^{5/4} + 6v^{1/4} + 3v^{-4})\,dv$

10 $\int (3v^5 - v^{5/3})\,dv$

11 $\int (3x - 1)^2\,dx$

12 $\int \left(x - \frac{1}{x}\right)^2 dx$

13 $\int x(2x + 3)\,dx$

14 $\int (2x - 5)(3x + 1)\,dx$

15 $\int \frac{8x - 5}{\sqrt[3]{x}}\,dx$

16 $\int \frac{2x^2 - x + 3}{\sqrt{x}}\,dx$

17 $\int \frac{x^3 - 1}{x - 1}\,dx, \quad x \neq 1$

18 $\int \frac{x^3 + 3x^2 - 9x - 2}{x - 2}\,dx, \quad x \neq 2$

19 $\int \frac{(t^2 + 3)^2}{t^6}\,dt$

20 $\int \frac{(\sqrt{t} + 2)^2}{t^3}\,dt$

21 $\int D_x \sqrt{x^2 + 4}\,dx$

22 $\int D_x \sqrt[3]{x^3 - 8}\,dx$

23 $D_x \int (x^3\sqrt{x - 4})\,dx$

24 $D_x \int (x^4 \sqrt[3]{x^2 + 9})\,dx$

Exer. 25–30: Evaluate the integral if a and b are constants.

25 $\int a^2\,dx$

26 $\int ab\,dx$

27 $\int (at + b)\,dt$

28 $\int \left(\frac{a}{b^2}t\right) dt$

29 $\int (a + b)\,du$

30 $\int (b - a^2)\,du$

Exer. 31–36: Solve the differential equation subject to the given conditions.

31 $f'(x) = 12x^2 - 6x + 1; \quad f(1) = 5$

32 $f'(x) = 9x^2 + x - 8; \quad f(-1) = 1$

33 $\frac{dy}{dx} = 4x^{1/2}; \quad y = 21 \text{ if } x = 4$

34 $\frac{dy}{dx} = 5x^{-1/3}; \quad y = 70 \text{ if } x = 27$

35 $f''(x) = 4x - 1; \quad f'(2) = -2; \quad f(1) = 3$

36 $f''(x) = 6x - 4; \quad f'(2) = 5; \quad f(2) = 4$

Exer. 37–38: If a point is moving on a coordinate line with the given acceleration $a(t)$ and initial conditions, find $s(t)$.

37 $a(t) = 2 - 6t; \quad v(0) = -5; \quad s(0) = 4$

38 $a(t) = 3t^2; \quad v(0) = 20; \quad s(0) = 5$

39 A projectile is fired vertically upward from ground level with a velocity of 1600 ft/sec. Disregarding ai find

(a) its distance $s(t)$ above ground at time t

(b) its maximum height

40 An object is dropped from a height of 1000 garding air resistance, find

(a) the distance it falls in t seconds

(b) its velocity at the end of 3 seconds

(c) when it strikes the ground

41 A stone is thrown directly downward from 96 feet with an initial velocity of 16 ft/sec. Find

(a) its distance above the ground after t seconds

(b) when it strikes the ground

(c) the velocity at which it strikes the ground

42 A gravitational constant for objects near the surface of the moon is 5.3 ft/sec^2.

(a) If an astronaut on the moon throws a stone directly upward with an initial velocity of 60 ft/sec, find its maximum altitude.

(b) If, after returning to Earth, the astronaut throws the same stone directly upward with the same initial velocity, find the maximum altitude.

43 If a projectile is fired vertically upward from a height of s_0 feet above the ground with a velocity of v_0 ft/sec, prove that if air resistance is disregarded, its distance $s(t)$ above the ground after t seconds is given by $s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$, where g is a gravitational constant.

44 A ball rolls down an inclined plane with an acceleration of 2 ft/sec^2.

(a) If the ball is given no initial velocity, how far will it roll in t seconds?

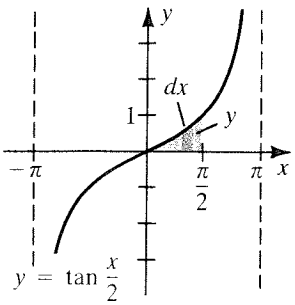
(b) What initial velocity must be given for the ball to roll 100 feet in 5 seconds?

45 If an automobile starts from rest, what constant acceleration will enable it to travel 500 feet in 10 seconds?

46 If a car is traveling at a speed of 60 mi/hr, what constant (negative) acceleration will enable it to stop in 9 seconds?

EXAMPLE 9 Find the area A of the region under the graph of the equation $y = \tan(x/2)$ from $x = 0$ to $x = \pi/2$.

FIGURE 8.28



SOLUTION The region and a typical rectangle of height y and width dx are sketched in Figure 8.28. As in Section 6.1,

$$A = \int_0^{\pi/2} y\,dx = \int_0^{\pi/2} \tan\frac{x}{2}\,dx.$$

We now make the substitution

$$u = \frac{x}{2}, \quad du = \frac{1}{2}\,dx$$

and note that $u = 0$ if $x = 0$ and $u = \pi/4$ if $x = \pi/2$. Thus,

$$\int_0^{\pi/2} \tan\frac{x}{2}\,dx = 2\int_0^{\pi/2} \tan\frac{x}{2}\cdot\frac{1}{2}\,dx$$

$$= 2\int_0^{\pi/4} \tan u\,du = 2\left[\ln\sec u\right]_0^{\pi/4}.$$

In this case we may drop the absolute value sign in $\ln|\sec u|$, since $\sec u$ is positive if u is between 0 and $\pi/4$. Since $\ln\sec(\pi/4) = \ln\sqrt{2} = \frac{1}{2}\ln 2$, and $\ln\sec 0 = \ln 1 = 0$, it follows that

$$\int_0^{\pi/2} \tan\frac{x}{2}\,dx = 2\cdot\frac{1}{2}\ln 2 = \ln 2 \approx 0.69$$

We will discuss additional methods for integrating trigonometric expressions in Chapter 9.

EXERCISES 8.4

Exer. 1–40: Evaluate.

1 $\int \frac{3}{4}\cos u\,du$

2 $\int -\frac{1}{5}\sin u\,du$

3 $\int \frac{7}{\csc x}\,dx$

4 $\int \frac{1}{4\sec x}\,dx$

5 $\int \frac{\sec t}{\cos t}\,dt$

6 $\int \frac{1}{\sin^2 t}\,dt$

7 $\int (\csc v\cot v\sec v)\,dv$

8 $\int (4 + 4\tan^2 v)\,dv$

9 $\int \frac{\sec w\sin w}{\cos w}\,dw$

10 $\int \frac{\csc w\cos w}{\sin w}\,dw$

11 $\int 3\sin 4x\,dx$

12 $\int 4\cos\frac{1}{2}x\,dx$

13 $\int \cos(4x - 3)\,dx$

14 $\int \sin(1 + 6x)\,dx$

15 $\int v\sin(v^2)\,dv$

16 $\int \frac{\cos\sqrt[3]{v}}{\sqrt[3]{v^2}}\,dv$

17 $\int \cos 3x\sqrt[3]{\sin 3x}\,dx$

18 $\int \frac{\sin 2x}{\sqrt{1 - \cos 2x}}\,dx$

19 $\int \sin x(1 + \cos x)^2\,dx$

20 $\int \sin^3 x\cos x\,dx$

21 $\int \frac{\sin x}{\cos^4 x}\,dx$

22 $\int \sin 2x\sec^5 2x\,dx$

23 $\int \frac{\cos t}{(1 - \sin t)^2}\,dt$

24 $\int (2 + 5\cos t)^3\sin t\,dt$

25 $\int \sec^2(3x - 4)\,dx$

26 $\int \frac{\csc 2x}{\sin 2x}\,dx$

27 $\int \sec^2 3x\tan 3x\,dx$

28 $\int \frac{1}{\tan 4x\sin 4x}\,dx$

29 $\int \frac{1}{\sin^2 5x}\,dx$

30 $\int \frac{x}{\cos^2(x^2)}\,dx$

31 $\int x\cot(x^2)\csc(x^2)\,dx$

32 $\int \sec\left(\frac{x}{3}\right)\tan\left(\frac{x}{3}\right)dx$