Plain-English Vocabulary List for Day 7

Extreme/Extrema

- Extrema is the plural of extreme.
  - It is not the supersoldier serum from Ironman 3
- An extreme is a high or low point on a graph.
- There are two types of extremes
  - **minimum/minima** (low point/points or valleys)
  - **maximum/maxima** (high point/points or hills)

Local/relative maximum/maxima and **local/relative minimum/minima**

- maxima is the plural of maximum; minima is the plural of minimum
- local and relative mean exactly the same thing
  - their opposites are **global or absolute**
- A local maximum is a point that is <u>higher</u> than the points around it; a local minimum is <u>lower</u> than the points around it.
  - Usually, this means that maxima are at the peak of a hill-shaped part of the graph and minima are at the bottom of a valley-shaped part of a graph
  - More unusual local minima/maxima occur at
    - sharp points (cusps)
    - discontinuities (breaks in the graph)
    - horizontal segments (in a horizontal segment, all points are <u>both</u> local maxima and local minima)
    - remember
      - if a point is higher than the points around it, then it's a local maximum
      - if a point is lower than the points around it, then it's a local minimum
- When calculus teachers say "minima" or "maxima," they usually mean "local," but they could mean both local and the global.
  - Yes, this is ambiguous and confusing
  - When students give an answer, they must specify whether it is a local or global (or use relative/absolute)
    - Yes, this is hypocritical
- Local maxima and minima cannot occur at endpoints

Global/absolute minima and **maxima**

- Global and absolute mean exactly the same thing
- A global minimum is lower than all other points of the function. A global maximum is lower than all other points of the function.
  - You can have more than one global minima if there are two points that are tied for the lowest point on the graph.
    - Think of the letter "**W**." The two lowest points are equally low and they are both lower than the rest of the letter, so they are both global minima (they are also local minima).
  - The same logic can be applied to global maxima. The two peaks of the letter "**M**" are both global and local maxima.
- Global extremes can occur at endpoints
- A point can be both a local and global extreme.

Increasing vs. decreasing functions

- A graph is increasing if it slopes upwards; a graph is decreasing if it slopes downwards
- Imagine if you're walking along a graph from left to right. The uphill parts would be increasing and the downhill parts would be decreasing.
- Increasing/decreasing behavior applies to sections of the graph rather than points, we typically use segment notation to specify which parts are increasing or decreasing.
    - i.e. "f(x) is decreasing on (0,4)" means "between x = 0 and x = 4, f(x) is decreasing"
        - note that this is an open set, meaning that f(x) is not decreasing at x = 0 nor is it decreasing at x = 4, but it is decreasing at all the points between x = 0 and x = 4.
- You can tell whether a graph is increasing or decreasing by looking at the first derivative.
    - f'(x) is positive if and only if the function is increasing
    - f'(x) is negative if and only if the function is decreasing

Critical numbers and critical points

- A critical number is a value of x where f'(x) is either 0 or undefined
    - critical numbers must be within the function's domain.
        - x = 0 is not a critical number of f(x) = 1/x
    - remember that all points (including critical points) have two coordinates (x and y)
        - the critical number is the x-coordinate of the critical point
        - a critical point has two coordinates whereas a critical number has only an x-value
    - a rarely-used term is a **stationary point**, which is only the critical points where f'(x) = 0
- All local maxima and local minima occur at critical points
    - not all critical points are local maxima or local minima

First Derivative Test

- A mathematical process for finding local and global extremes of a function without graphing.
- The steps are
    - Find the first derivative
    - Find the critical numbers
    - Set up a sign chart: a number line that includes all the critical numbers
    - For each interval
        - Pick a test value, which can be any value within the interval
            - since you can choose any test value, you should try to pick an easy one
        - Calculate f'(x) for the test value
            - If f'(x) is positive, then the function is increasing on the entire interval
            - If f'(x) is negative, then it's decreasing on the entire interval
    - Classify the critical points as local maxima, local minima or neither
        - increasing to decreasing → local max
        - decreasing to increasing → local min
        - no change of sign → neither
    - Find global extremes
        - Calculate f(x) for each of your local extremes
        - Calculate f(x) for each of your end points
        - The point that produces the highest value of f(x) is the global max; the point that produces the lowest value is the global min.